Hall Ticket Number:

Code No. : 1213 I

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. I Year II-Semester Examinations*, July/August-2016

Mathematics-II

Time: **3** hours Max. Marks: **70**

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Part-A (10 × 2 = 20 Marks)

1. If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $(\bar{u}.\nabla)\bar{r} = \bar{u}$
2. Find a unit normal vector to the surface $xy^3z^2 = 4$ at the point (-1,-1,2).
3. Solve $(xy^3 + y)dx + (2x^2y^2 + x + y^4)dy = 0$
4. Solve $y = xp - p^3$ where $p = \frac{dy}{dx}$.
5. Solve $(D^3 - 7D - 6)y = 0$
6. Find the particular integral of $[D^2 + 2]y = x^2$
7. Define ordinary and singular points.
8. Show that $P_4(x) = \frac{1}{8}[35x^4 - 30x^2 + 3]$
9. Define gamma function
10. Prove $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2 = \frac{2}{\pi x}$

Part - B (5 × 10=50 marks)
(All bits carry equal marks)

11. a) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$

    b) Using Green`s theorem in the plane for $\oint_C (x^2 + y^2)dx + (y + 2x)dy$ where C is the boundary of the region defined by $x = y^2$ and $y = x^2$.

12. a) Find the general solution of the Riccati equation $y' = 3y^2 - (1 + 6x)y + 3x^2 + x + 1$. If $y = x$ is a solution of the differential equation.

    b) Find the equation of the family of orthogonal trajectories of the family of circles which passes through the points (2,0) and (-2,0).

13. a) Solve the initial value problem $y'' - 2y' + 3y = 0$ with $y(0) = 1, y'(0) = 0$.

    b) Find the general solution of the equation $y'' - 2y' + 2y = e^x tanx$ using the method of variation of parameters.

14. a) Prove that $(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$

    b) Show that $(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x)$

Contd...2

15. a) Show that $\beta(m,n) = \dfrac{\Gamma m \Gamma n}{\Gamma m+n}$

b) Show that $J_3(x) = \left[\dfrac{8}{x^2} - 1\right] J_1(x) - \dfrac{4}{x} J_0(x)$

16. a) Find the directional derivative of $f(x,y,z) = x^2 + y^2 + 2z^2$ at the point (1, 1, 2) in the direction of $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

b) Solve the differential equation $(D^2 + 4)y = x \sin x$

17. Answer any **two** of the following:

a) Solve $\frac{d^2y}{dx^2} + x^2 y = 0$ in series about x = 0

b) Prove that $\int_{-1}^{1} P_m(x) P_n(x) dx = 0, \quad m \neq n$

c) Prove that $\gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

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